Resisting the desire for the unambiguous: productive gaps in researcher, teacher and student interpretations of a number story task

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Abstract
This article offers reflections on task design in the context of a Grade R (reception year) in-service numeracy project in South Africa. The research explores under what conditions, and for what learning purpose, a task designed by someone else may be recast and how varying given task specifications may support or inhibit learning, as a result of that recasting. This question is situated within a two-pronged task design challenge as to emerging gaps between the task designer's intentions and teacher's actions and secondly between the teachers' intentions and students' actions. Through analysing two teachers and their respective Grade R students' interpretations of a worksheet task, provided to teachers in the project, we illuminate the way explicit constraints, in the form of task specifications, can be both enabling and constraining of learning. In so doing we recast this 'double gap' as enabling productive learning spaces for teacher educators, teachers and students.

Key words: numeracy task design; task specification; number stories

1. Introduction

In this article we focus on the enablers and constraints that arise in relation to teacher and learner use of tasks designed by a researcher/teacher educator (1st author) to foster awareness of number in the reception year (Grade R ages 5-6 years) of primary schooling. The research questions under what conditions, and for what learning purpose, a task designed by someone else may be recast and how varying given task specifications may support or inhibit learning as a result of that recasting. The article is based on an enactivist analysis of two teachers and their Grade R students' responses to a worksheet type task, provided to them in an in-service numeracy project. We begin by setting the context of the teacher development project in which the task was carried out. We then provide an explanation of the task and locate our approach within broader literature on task design. We explain the enactivist methodology used and set out the findings, which illuminate opportunities and challenges in relation to the extent of a desire for constraints and explicitness in both task design and task enactment.

While research in task design has pointed to a gap between teacher and learner intentions/interpretations in relation to tasks, here we illuminate a potential double gap occurring when a task is designed by a teacher educator/researcher, emerging between (i) the intentions and suggested specifications of the task designer and the enacted task specifications (interpreted by teachers and communicated by them to students) and (ii) the teachers' interpretations and stated specifications and their students' enacted response to the task. In each stage of the 'passing on' of the task, verbal and enacted specifications are communicated where re-interpretations may differ from original intentions.

While, as we have just done, this situation is often framed in terms of 'gaps', in this paper we suggest that differences of interpretation can become productive learning spaces when there is the possibility of reflecting explicitly on them, rather than succumbing to a desire for the fantasy of the unambiguous. This learning is evident in the many 'Aha' expressions of the first author (as task designer and teacher educator) during her interviews with two teachers who used the task with their students.

From these reflective interviews the desire for explicit and unambiguous task instructions (and layout) is set against the learning gains for teacher educators, teachers and students when the task is left open to various interpretations. The data herein highlights the way in which different teacher specifications of the same 'worksheet' task lead to different enacted tasks and thus learning opportunities. In this respect it may be more appropriate to ask under what conditions, and for what learning purpose, might we recast a task designed by someone else and how might the varying specifications given to students support or limit the learning that results from that recasting?

2. Task design

Watson and Ohtani (2012, p.4) define a mathematics task as "anything that a teacher uses to demonstrate mathematics, to pursue interactively with students, or to ask students to do something". We adopt this definition and hence view the 'worksheet' activity herein as an example of a mathematical task, however Watson and Ohtani go on to say that a "Task can also be anything that students decide to do for themselves in a particular situation" (p.4); some students may enact a task in a way which is not aligned to the skills intended by the teacher.

One of the starting points for this Special Issue is the recognition that teacher intentions and student experiences of tasks can be widely different. Margolinas (2005) pointed to the bifurcation of perspectives, expectations and experiences of the teacher compared to the students. The teacher is, usually, the expert and there is a significant question (Mason et al., 2005, p. 131) around how an expert's awarenesses might become available to students. Awarenesses can get translated into tasks for the learner that do not lead to those same awarenesses. Chevallard (1988) raised the problem of moving from the knowledge used in a sphere, such as mathematics, to the knowledge to be taught, a phenomena he labelled the 'didactic transposition'. The issue again being how awareness within a sphere might be translated into actions in the classroom that can lead to those same awarenesses.

Tahta (1980) distinguished 'outer' and 'inner' aspects of tasks, i.e., what is made explicit by the teacher and the relationship or awareness the teacher hopes students will gain. The more the desired behaviours in students are specified, the less these behaviours are likely to emanate from students' own awareness. Another way of stating the issue is that shifts in noticing or attention (Watson and Mason, 2007) cannot reliably be brought about through words. The situation might be compared to the expert and novice piano tuner (with thanks to Markku Hannula for this anecdote). An expert tuner will hear differences in tone that are not available to the novice and asking: 'can't you hear the difference?' is probably not useful. What is required (on the part of the novice) is a shift in perception of the situation. Pointing to this required shift (making it explicit) is not the same as the novice experiencing that shift. And, as Coles and Brown (2016) state, 'no matter what we do as teachers, we cannot make that shift or transformation happen for learners' (p.151).

The major traditions of task design have highlighted the difference between having a shift in awareness pointed out and experiencing that shift. Cuoco et al.'s (1996) curriculum based on 'habits of mind' (such as "students should be conjecturers") attempts to get around the problem by suggesting teachers encourage ways of engaging in mathematics that will make it likely students experience transformations in awareness. As teachers, if we find ways to encourage students in making conjectures, it is clear that the spark of insight (to make the conjecture) must come from the student. This way of addressing how to make available to students expert awareness is to focus teaching at a 'meta-level' to the mathematical awarenesses that are desired.

In the design research tradition (DBRC 2000), researchers provide teachers with high quality tools that have been through cycles of testing and adapting, to solve particular pedagogical problems (which might be about anything from a particular item of content up to an entire curriculum). The approach to making available expert awareness is to use the expertise of researchers, and the feedback from trials, to design tasks where there is evidence that students *do* gain the intended awarenesses. What can be occluded in descriptions of this approach is the mediation of tasks by teachers.

The theory of Didactical Engineering (Artigue and Perrin-Glorian 1991) shares similarities with both approaches to the expert-awareness-issue described above. Similar to Cuoco et al. (1996), importance is placed on the meta-level of students making their own discoveries within mathematics although with a different emphasis in terms of how teachers might bring about such a learning environment. And, in line with Design Research, there is an emphasis on a cyclical process of a priori analysis, classroom testing and a posteriori analysis. In contrast to Design Research there is a more developed and more tightly specified role for the teacher with, perhaps, a more limited range of contexts in which the approach might be applied but where the teacher role is more visible.

The final tradition we review briefly is the Realistic Mathematics Education (RME) programme (Van Den Heuvel-Panhuizen 2003). This tradition can be viewed as having a theoretical and pragmatic stance on how expert awareness becomes available to students, which is through first engaging students' intuitive understanding of an imaginable context, then supporting a process of mathematising these intuitions into progressively abstract models that ultimately result in formal mathematical systems that are imbued with meaning through retaining their link to students' original intuitions.

Through all these traditions we interpret one common aim as the desire to reduce 'gaps' between researcher intention, teacher intention and student activity (e.g., through cycles of testing and refinement) and to be able increase the likelihood of specific expert awareness becoming available to teachers and to students. Our research question, by contrast, explores under what conditions, and for what learning purpose, a task designed by someone else may be recast (by teachers or students) and how varying given task specifications may support or inhibit learning as a result of that recasting.

In this paper, the task moves from the task designer (teacher educator/researcher) to the Grade R teacher and from the Grade R teacher to her Grade R learners. We note that while some aspects of the 'expert's awareness' becomes available to teachers in different ways, and some aspects of teacher awareness become available to learners in different ways, several aspects are lost or transformed along the way. Rather than view these losses as 'gaps' we prefer to see them as potential spaces for learning, as we illustrate below, first in theory and then via empirical results.

3. The Context

South African mathematics education is widely noted for performing below national expectations and regional and international averages (Graven, 2014). The Department of Basic Education's (DBE) Annual National Assessments (ANA), consistently point to poor mathematics results with only 3% of learners achieving 50% or more in the last written ANAs in 2014 (DBE, 2014). Widespread evidence of an absence of number sense in FP learners is a critical concern not sufficiently addressed (Graven et al., 2013). A focus on producing (and awarding marks for) 'the right' answer irrespective of whether methods used are efficient or appropriate masks the challenges of the lack of Foundation Phase (FP: Grade R-3 ages 5-9) competences. Weitz & Venkat (2013) demonstrate that students who pass the Grade 1 and 2 ANAs, when assessed according to the strategies used, have not progressed beyond the most basic levels of reasoning according to Wright et al.'s (2006) Learning Framework in Number.

Curriculum policy has since 1997 included Grade R as the first year of the Foundation Phase (Grade R-3) thus connecting it to formal schooling. While Education White Paper number 5 of 2001 stated that Grade R should be offered mainly at schools, rather than separate early childhood centres, there are still schools where it is not offered and still separate centres who offer it (DBE, 2011). Furthermore there are large disparities in the quality and qualifications of Grade R teachers (many are un or under qualified) and pre-service teacher training is not well developed (DBE, 2011). While the DBE (2011a) acknowledges that teacher development is essential for enabling quality Grade R teaching, there is little evidence of support for teachers. Furthermore numeracy is often under represented in FP teacher education programs where there is a need for *specialist* elementary mathematics programs to strengthen early learning (Graven & Venkat, 2017). Thus it was considered important to establish a supportive community for Grade R teachers in which numeracy learning would be foregrounded, though integrated with other aspects of the Grade R curriculum.

The South African Numeracy Chair Project (SANCP) began in 2011 at Rhodes University with the incumbent Chair (1st author) mandated to work at the interface of research and development to find sustainable ways forward to the challenges of numeracy education in South Africa. SANCP is currently running a Grade R teacher development programme. This was preceded by a Grade 3-4 programme (see Graven, 2016) and is to be followed by a Grade 1-2 programme). The focal task in this paper was developed for use within the Grade R in-service teacher development program called Early Number Fun (ENF), which began in April 2016 and has 33 Grade R teachers from 17 schools in the broader Grahamstown area. These teachers partner with researchers and teacher educators in the SANCP to collaboratively find ways to strengthen numeracy learning, particularly for learners in resource constrained contexts. ENF meets monthly for afternoon sessions revolving around a series of key themes (such as an integrated or narrative approach to developing number sense) and key resources (such as bead-strings, dice, flash cards).

Teachers investigate how various research-informed resources provided in ENF work in class and feed back experiences, adaptations and extensions at the start of each ENF session. Teachers also share video recordings and photographs of their adaptations of resources/activities. Adaptations are made before resources are placed freely available for wider teacher use. Two ENF participants are education department specialists who then share adapted ideas and resources with teachers more widely in their departmentally mandated teacher support work. The worksheet task in this paper was shared in the 2nd ENF session for use after a series of activities based on a '5 monkeys in a tree' book.

3.1 The suggested activity sequence

'5 monkeys in a tree' was the first number storybook given to teachers. The story begins with five monkeys in a small tree and no monkeys in a big tree. Each subsequent page has one monkey jumping from the small to the big tree. Page 2 of the story is given below. The full story can be found on the SANCP website[1].

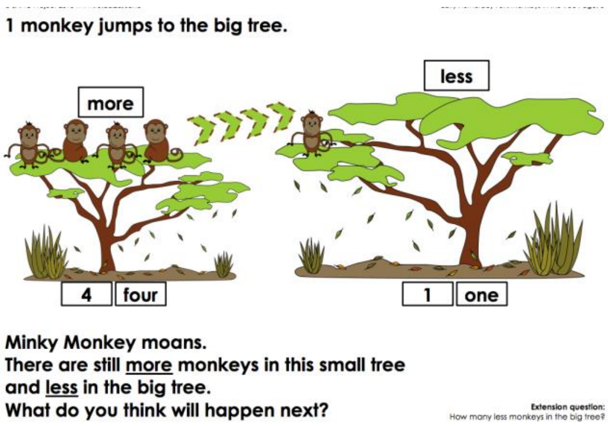

Figure 1: Page 2 of '5 monkeys in a tree'

The activity sequence was introduced to teachers (2nd ENF session 17th May 2016). A demonstration of the story sequence (read story; read again with students acting out; students individually re-enact the story using finger puppets) with two learners was done in view of the teachers. The worksheet was not demonstrated as the children by this stage were becoming tired. Teachers were instead shown an example of what two students produced in the pilot. I.e. 5-0-5; 4-1-5; 3-2-5; 2-3-5; 1-4-5; and 0-5-5 written in the 6 rows of block-block-circle on a hand drawn version of Fig. 2 below but without connector lines and arrows.

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[1] http://www.ru.ac.za/sanc/teacherdevelopment/earlynumberfungrader2016-2017/

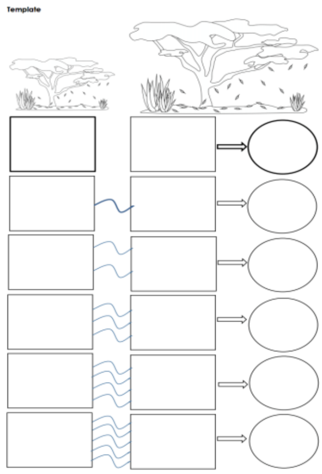


Figure 2: Monkey story written task

Teachers were given the books, flashcards, puppet templates and a set of worksheets. A summary of the suggested activity sequence is given below. The books were given in the languages of instruction across ENF schools (English, Afrikaans and Isi Xhosa). Teachers were encouraged to:

- use dialogic reading to: point attention to the number of monkeys in each tree and the number of monkeys altogether on each page; ask questions about which tree has more or less and, ask learners to predict what happens next.
- have students act out the story with laminated flash cards to describe and compare the number of monkeys in each tree at each stage. I.e. 5 students get to act out the monkey jumps while others choose flashcards of 'more' 'less' and numeral and number-word cards (e.g. 4-four) to represent quantities 'in each tree' at each stage.
- have each learner colour in, cut and tape 5 finger puppets and then use these to re-enact the story on their fingers.
- after these activities, give the worksheet (Fig. 2 above) to students thought to be ready for written representation of the story. It was suggested they tell learners to use the blocks to tell the story about the monkeys in each tree at each part of the story in whichever way they choose. It was emphasised that learners need not use numerals. It was noted that the circles are for the total monkeys in both trees.

3.2 The rationale and intentions for the activity sequence and written task

The first author's passion for developing 'number' stories emerged from her experience of reading the early reader book 'Ten apples on top'[2] to her children from age 2 till 8. She loved the mathematical conversation and sense making that her girls engaged in when she read the story to them using dialogic reading (Doyle & Bramwell, 2006)[3] with some acting out. Recently local research focused on a narrative approach to working with early additive reasoning problems (Roberts 2016; Takane, Tshesane & Askew, 2017) supports our assumption here that number stories can support early number learning. Since ENF aimed to develop Grade R teacher

2 By Theo LeSieg & Roy McKie. Published by Beginner Books.

3 Dialogic reading involves multiple readings and conversations about books with strategic questioning and responding to children (Doyle and Bramwell, 2006).

identities as integrators of numeracy, literacy and life skills, such number stories helped communicate that the numeracy focus was not at the expense of literacy. Furthermore, the sequence of related activities connected with the three stages of learning foregrounded in the Grade R curriculum document (DBE, 2011a, 14), i.e. kinaesthetic ('experience concepts with the body and senses' through re-enactment of stories); concrete ('using concrete objects for modelling' through using finger puppets) and paper and pencil representation ('semi-concrete representations using drawings' through the written task).

The mathematical intentions of the activity sequence as communicated to teachers were to enable learners to engage with: context and object bound counting 1-5 and calculating (1 less/more); numeral and word recognition (0-5); comparative language use and word recognition (more, less); a patterned sense of bonds to 5 (i.e. 5-0; 4-1; 3-2; etc.), and use of written forms to represent the changing 'number of …' at each stage of the story.

4. Methodological framing and initial analysis

As stated in the Introduction, our overall methodological stance is enactivist (Reid and Mgombelo, 2015). Enactivism entails a view of cognition as arising through interaction. Through interaction, organism and environment 'co-evolve' in a process that alters the very structure of each other. An organism's structure is the particular set of relations of all the internal components that make it what it is (including, for example, neural patterns). There is therefore a "structural coupling" (Maturana & Verden-Zoller 2008, p.26-7) of organism and environment and what counts as having value is continually under co-construction (Thompson and Stapleton, 2009). However, the way a living organism responds in any context is determined by its structure, not the context itself. The environment (which includes other organisms) can only trigger a response, how an individual acts is always a function of his or her history.

For enactivist research, distinctions and differences play a significant role, since perception has its basis in the noticing of distinctions, or differences that "make a difference" (Bateson, 1979, p.27). From an enactivist stance, perceiving, acting and knowing cannot be distinguished, "all doing is knowing, all knowing is doing" (Maturana and Varela, 1987, p.27). When a researcher designs a task that a teacher will use and students will enact, it is inevitable that the sense made of the task will be different for each person and based on their entire histories of interaction in the world. Rather than frame these differences as undesirable 'gaps' that could be overcome, enactivism commits us to an alternative view. Differences in responses to a task result from differences in structure and experiences, but through engaging in exploration of such distinctions it is possible to come to perceive a situation differently and this, from an enactivist perspective, is equivalent to learning. Through constraining what others do, it might be possible to convince ourselves that words or instructions can be heard without ambiguity. We believe this is never possible and that all successful communication necessitates a sharing of differences and that, when given space, such differences are productive. A pluralism of methods are possible within enactivist research where what is significant is how data collected is used; i.e., to provoke new distinctions for researchers and participants. Methodologically, enactivism commits us to a systematic search for pattern (Coles, 2015), taking multiple views of data and a cyclical process of data collection and analysis, leading to further data collection and analysis (Reid, 1996). The role of the observer is acknowledged in all communication (Maturana and Varela, 1987) and hence methodologically we are committed to the view that actions and language give us access to the distinctions made by observers.

Eleven teachers brought a sample of their learners' worksheets (ranging from 1–11), which they shared in ENF group reflections. Other teachers, who had not used the worksheets in class, brought photos of learners participating in other activities in the story sequence. All worksheets were copied to enable reflection on them for future adaptations (with permissions granted) and returned to teachers in the same session. Following this, an initial textual analysis was done to identify types of learner responses. This was done in a grounded way and so a new category was generated for each response that did not fit into an existing category. These categories were then refined to combine similar categories and to create sub-categories within these. Furthermore when looking across the sets of worksheets brought by each teacher there were two noticeable pairs of differences – sets of worksheets that were clearly marked with ticks and crosses at the end of each row and those that were not; and sets of worksheets where there were different ways of working within a set and others where all worksheets were similar with either numerals only or pictures only. The distinction noticed (student worksheets 'the same' or student worksheets 'different') provoked us to seek multiple views (data analysis leading to further data collection). We therefore chose two teachers, one from each category with the intention of exploring their observations about what had happened in their classrooms.

We focus this paper on the interpretations and resultant completed worksheets of two ENF teachers, Anne and Thandi, and their 27 and 22 Grade R students' worksheets. The two teachers and their sets of worksheets were chosen because they i) capture the two noticeable differences of broad approaches above and ii) they were the only two ENF teachers who brought *all* completed worksheets of their students to the 3rd ENF session. Anne and Thandi's classes are in low fee paying government schools. The medium of instruction for both schools is English. Both schools have a mix of learners from middle class to poorer backgrounds. 18 of Anne's and all of Thandi's learners are home language isi-Xhosa speakers.

Following initial reflection on the range of student worksheets, two stimulus recall interviews were conducted with Anne and Thandi - but rather than video as a stimulus for recall (as suggested by Lyle, 2003) the teachers' student worksheets were used as a stimulus. As Lyle (2003, p.861) explains stimulated recall enables investigation of cognitive processes 'by inviting subjects to recall, when prompted by a video sequence, their concurrent thinking during that event.' Thus copies of the two teachers' student worksheets were used in the interviews aimed at gathering more detailed information as to how the activity was introduced and how learners engaged with the task. Both interviews were audio recorded and transcribed verbatim with all names changed other than the interviewer (Mel – 1st author). These interviews allowed us multiple views of the data as we were able to contrast our own distinctions with those made by the teachers.

Prior to the interviews, one further stage of analysis was undertaken on the worksheets themselves to inform the interview conversation. The worksheets were organised into types by the first author so that teacher interpretations of student interpretations for each type of response could be gathered – stimulated by re-viewing the student work in each type. For Anne, the three types of scripts were: numerals only; tallies/pictures only; scripts with a combination of the above. For Thandi, the two types of scripts were: scripts with numerals and dots (for bottle tops) and scripts with numerals only.

5. Interview data and further analysis

We offer here a selection of data and a commentary, related to Anne and Thandi. Our overall research interest was to understand more about the differences in their interpretations of the task. We present, therefore, all those sections from the interview data related to differences in interpretations, i.e., differences we notice between the researcher and teacher; between the two teachers; or, between the teacher and students. We have chosen to report the clearest examples from our data that draw out distinctions and attune ourselves to patterns in the data. We present transcripts followed by our commentary related to distinctions in interpretation of the task.

5.1 Teacher introductions to the written task - explicitness and constraints

Anne and Thandi gave the written task after just under four and three weeks respectively of working with the activity sequence. Thandi gave students bottle tops for representing monkeys, which they then moved from hand to hand to model each step of the story to correspond to filling in each row of the worksheet. Similarly Anne used finger puppets placed on the worksheet to model what happened in each step of the story for six learners who struggled to make sense of the worksheet.

In Anne's stimulated recall interview (29 Aug), she explained her introduction of the worksheet to her class as follows: [… indicates some text missed out for ease of reading]:

> Anne: So when we sat down I said to them here's the small tree, here's the big tree (pointing to trees on worksheet) now we've got to see how many monkeys were there in the beginning in the small tree and we've got to move them over to the big tree just like the story did, I want you to show me how you are going to do that.
> Mel: Okay
> Anne: I never for once told them what to draw…Yes lets tell the story now, who was in the small tree? when did they move to the big tree? and how did that happen? I did walk around and I told my helper don't tell them what to do, don't tell them its wrong because I don't want them to at this stage…I absolutely gave them no instructions as to draw a monkey, draw a dot, draw a number at all.

In Thandi's stimulus recall interview (28 Sep) she indicated that only 22 of her 40 learners were given the worksheet as she only had 22 copies. She worked with the learners on the mat in groups of eight and seven. She selected a range of students based on her perception of their strength. "From that twenty-two, I gave to five that is excellent; five that is good; five that is average and five from the weak so that I could see their strengths and weaknesses". She explained that she introduced the worksheets to each group as follows:

> Thandi: I started to introduce them by the mat, showing them the blank paper and saying you are now going to draw for me and write the number and because they were able to write the numbers

perfectly by May up to six … and I even introduced plus and minus at that time and like if you see some of them now they are able to write the sums.
Mel: So basically you said to them write the numbers and draw the bottle tops?
Thandi: Yes, as you understand. While they were sitting in that group of eight I sat with them and I said okay count your bottle tops, once you are done I say okay lets move another one then they write and then I say again move, move.
Mel: Okay so they acted out with the bottle tops each step?
Thandi: Yes yes… I first did it in Xhosa and then in English it took about twenty minutes because they hold the bottle tops and I say which must move? I tell them I say 'how many in the small umbrella?'[4] They say 'five' so I say 'okay draw five', then they say in the big umbrella 'there is zero', so I say 'draw zero', then I say 'five plus zero' and they say 'five' so I say 'write five', then they say 'I have four', and then a child has moved one, and I say 'its four plus one', they say 'five', I say so write 'five'.

Thus in terms of the introduction of this task from designer (Mel) to teachers (Anne and Thandi) and from the teachers to their students key distinctions emerge in the task specifications that result in openings for multiple interpretations and enactments of the task – some cohere closely to the task intentions and others depart from these as will be seen when analysing teacher interviews based on student worksheets. It is useful to note the implicit specifications built into the 'outer' task through the layout and visual mediators in the design by Mel. That is, small and big trees above the columns of blocks but no visual mediator above the column of circles; six rows of two blocks with one circle to correspond to each page of the story; the order of small tree on the left and big tree on the right; the format of columns of blocks under each tree with a column of circles at the end (without a reference picture above); the sets of wavy lines between blocks to represent the cumulative number of jumps at each stage of the story, and the arrow lines from the blocks under the big tree to the circles. Teacher and student interpretations of these features do not always cohere with Mel's intention for them as will be seen in the analysis. In the table below, we summarise the verbal task specifications given to teachers by Mel as teacher educator in ENF and Anne and Thandi's verbal specifications given to their students. We trace these differences through the rest of our data in the next section.

Table 1: Summary of verbal specifications in passing the task on to others.

Mel to ENF teachers	Anne to students	Thandi to students
Use these worksheets if you think your students are ready after other story activities. Ask learners to use the worksheet to retell the story at each stage. Blocks are intended for the number of monkeys in each tree and circles for the total number in both trees at each stage. Students can use any form of representations (e.g. tallies/pictures) - not just numerals.	Tell the story using this worksheet. Deliberate in not giving further specifications.	Students were told to use bottle tops to show what happened then Thandi would with questions and instructions take students through filling in each row: e.g. Stage 1 'How many here?' 'So draw five' 'How many here?' 'So draw zero' 'five plus zero?' 'So write five'.

The table shows several differences in the task specification of Mel, Anne and Thandi. The most notable contrasts being (i) Anne's deliberate instruction to 'use the

[4] A similar book '5 children under umbrellas' was later given to teachers hence the interchange in talk of trees and umbrellas.

task to tell the story' and Thandi's step by step instructions to learners of what to write based on remodelling the story with stones in their hands (ii) Mel's suggestion that students be allowed to use any form of representation, mirrored by Anne, and Thandi's insistence on writing numerals. Anne's injunction to her helper 'don't tell them what to do' suggests that 'not telling' is a different practice, for that helper, to the norm in her classroom.

5.2 Results and analysis of student worksheets

We categorised student worksheets to inform questions Mel would ask during interviews and, for Anne, the three types of scripts were: numerals only; tallies/pictures only; scripts with a combination of the above. Within these categories Mel asked questions about the sub categories in these in relation to variations in the: correctness/incorrectness of representation of the quantification of monkeys at each stage; attention, or lack of, to the connector lines between the blocks and, what was written in the circles. For Thandi, two types of scripts were: scripts with numerals and dots (for bottle tops) and scripts with numerals only. Within the first group Mel asked about the variation where eighteen scripts began with 5-0 in numerals and dots in the top row while two began with 4-1 in the top row. All twenty-two scripts had only the numeral 5 in each of the six circles on the worksheet.

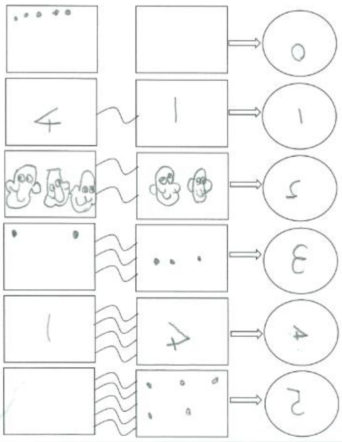
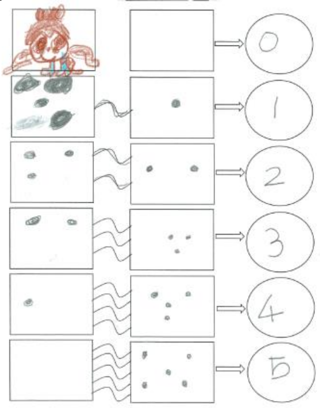
We offer, below, samples from the data and the teachers' reflections, followed by our own commentary on distinctions and differences in interpretation of the task (across researcher, teachers and students).

5.2.1 Interpretations of Anne's student worksheets

Analysis of Anne's interview focuses on the three categories of student interpretations, as elaborated below.

Category 1: Independent interpretations aligned to key task intentions
The 16 worksheets in this category all showed mostly correct representations of the changing number of monkeys at each stage. We say mostly because two students made single errors (one began with 5-1 in row 1 and another repeated 3-2 in the 4th row and one did not complete the 6th row). There were some variations in terms of: ways of representing the monkeys (pictures, dots, numerals, a mixture); student attention to the connector lines (evident in some drawing over these and some drawing over these their own single jump line at each stage); and, interpretations of what the circles represent. Based on Anne's interview comments these 16 worksheets were considered by us to indicate independent student interpretations of the blocks part of the task that were aligned to the intentions/specifications of Mel and Anne. However in terms of the circles aspect of the task no student in Anne's class wrote 5 in every circle and so all students' interpretations of what was required in the circles departed from Mel's intention for circles to represent the total monkeys in both trees at each stage.

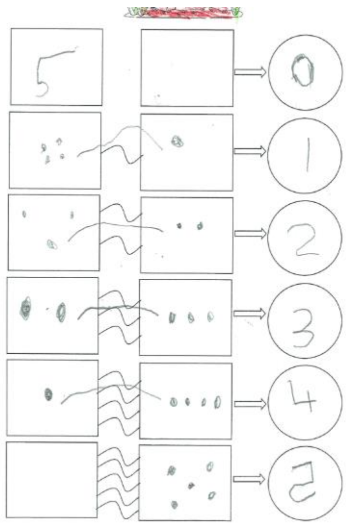
For these 16 students it would seem the task (excluding the circle aspect) and the relatively open instruction from Anne was sufficient to sensitise them (Mason & Watson, 2007) to what to attend to and notice mathematically in each part of the story (i.e., as intended in the design, the systematic changing quantities of monkeys in each tree by one each time) as the story unfolds. Below are examples of two students' worksheets that show different representations, which correctly retell the changing quantities of monkeys in the trees. The related interview reflections indicate Anne's interpretation of student interpretation of, and thinking in completing, the task.

	
Anne: So this is the sort of thing that would go through her mind, now she and Vee sit next to each other and you can see there has been no copying whatsoever, so she decided no I'll do dots, now I'll change to the number, and then I'll put some monkeys and then I'll go back to whatever.	Anne: This is an interesting thing I think once she drew that (points to the monkey drawn) she realised she couldn't draw anymore so, she had drawn it too big, it had taken up the whole box. 'So actually I can't draw five so I'm going to dots' because the instruction was show the monkeys jumping from the small tree to the big tree so I think she thought I must draw monkeys but then realised but my monkey is so huge I can't draw another four so I think I'll just go onto the next ones.

As researchers, we would not want to conjecture what these children were thinking, as they responded to the task, but we note their representations, although not consistent, are generally unambiguous.

In terms of the lines connecting the first two columns of blocks (connector lines) five of these 16 worksheets showed learners paying attention to these (intended by Mel as designer to cumulatively represent the number of jumps that occurred by each stage). This was seen in three students drawing over the first few sets of connecting wavy lines (as in Mpho's above) and two students drawing over all sets of lines. On the other hand two of these 16 worksheets showed learners drawing their own curved 'jump' line between blocks (seemingly ignoring the existing lines and instead representing the one jump that took place at each stage). The worksheet below shows how these students re-represented the jumps as only *one* jump at each stage - representing this with a jump arc from the drawn picture to the drawn picture. These students ignored or rejected (as Anne indicates for Jaya below) the task design's cumulative use of connector lines for the total jumps that happened by that stage. In the design the lines are historical versus these learners indication of one jump in the present tense of each stage. Of note is the 'aaah' moment Mel has as she becomes aware that the connector lines likely constrained many students from their own sense making of how the jumps happened and that her connector lines were a poor representation of the pathway of a jump.

Figure 3: Jaya's worksheet



Anne: She's ignored your lines and one of the things they enjoyed in the acting was the monkey *jumped* to the other tree and so when they were doing it themselves they liked to jump so I think what she's decided is she's going to show you jumping
Mel: yes
Anne: but it wasn't two monkeys jumping it was only one every time
Mel: well one jumping path
Anne: because when there were four *one* jumped, when there were three *one* jumped, when there were two *one* jumped
Mel: Aaaaah
Anne: One jumped when there was one - *one* jumped. There was never *two* at a time, there was never *four* at a time!
Mel: Aaaah
Anne: So she looked at this and thought I'm drawing the monkey that's jumping, each time another monkey jumps
Mel: Yes!
Anne: so she ignored your lines because she said four monkeys didn't jump - one jumped
Mel: and did she say that to you?
Anne: I can actually remember her saying that that's the monkeys jumping (pointing to the drawn arc) – ya so what were these lines (pointing to the set of set four curvy drawn lines) there for? Because four didn't go at the same time…
Mel: ok fantastic! So what's interesting is that her line shows that's the path of a jump my lines that's not a path of a jump mine dip (both laughing)
Anne: yes they don't do that
Mel: mine bounce (pointing to the bottom curve of the squiggle lines - laughing)
Anne: as far as she's concerned that's (her curve line) a jump
Mel: Mine were a kangaroo
Anne: yes exactly she hasn't done a straight line she's done a jump
Mel: She's done a jump line!
Anne: She has actually portrayed your jump each one…
Mel: yes so it makes me think that the lines might have just got in the way I should have just left them out.

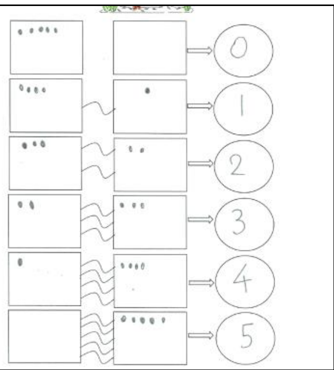
We are struck by the layers of distinctions apparent in this interview section. Jaya's worksheet indicates that she was paying attention to the dynamic in the story. At each stage of the story, one monkey jumped and she represented this change with her lines. Mel is helped to make a new distinction (for her) in terms of worksheet design, raising the possibilitiy of now making a choice between representing change, representing cumulative change or leaving those options open to the learner.

In terms of the circles, twelve of these sixteen students wrote the numerals 0-5 sequentially from top down in each circle. Anne interpreted this as students thinking they needed to write, in another way, what they had written in the last block of each row because the worksheet had an arrow from only each right hand block to each adjacent circle.

> Anne: So they've gone what was in the box next to them in the big tree
> Mel: Aaaah so theve gone what was in the box in the big tree. I get it now its (the arrow line) asking what was in the big tree?
> Anne:.. so they look at so that (the arrow) has come from the big tree (points to the arrow from the box below the big tree)…Ya [students thought] ‘I can’t have the same because that's different to that one (circle not a block) so that must be different’
> Mel: fascinating.

The four other students: drew pictures of monkeys in each (1); put a mix of dots, numerals and/or pictures in each circle (2); or drew one picture (of trees, cars and people) in each circle. For these students the circles were interpreted as not specifically representing the monkeys in the big, or both trees, at a particular stage but rather as an opportunity to draw something related to the story.

Category 2: Teacher support needed for interpretation
Anne indicated that five students needed support of concretely using finger puppets laid on the rows of blocks to connect what they should write to what happened at that stage in the story. Of these five only one learner did not manage to transfer the corresponding number of dots/tallies in relation to the puppets laid down and two students (see example below) managed on their own eventually (i.e. no longer needed support for the last rows):

	Anne: She understood with help… and then she managed to actually have ‘aaah’ light bulb moment, ‘I know what you’re looking for now’ and a lot of her is quite unsure as in she didn't want to make a mistake Mel: Okay and so did she have the puppets here to generate the first two (rows)? Anne: Yes and then she used her own puppets to generate the rest so she did do it but by herself with the rest, ya. But it just needed that extra bit of help and something visual to see what we were talking about …

Category 3: Interpretation that copying others can produce required written response
Anne noted six worksheets as copied from other specific learners. Comparing these in the stimulus recall interviews it seemed there was clear evidence of this. For example in some cases the exact positioning of dots or the same incorrect orientation of numerals were seen on worksheets of learners seated next to or opposite each other.

> Anne: This could also be from copying (takes two scripts of learners who sit next to each other)
> Mel: So you think that's why this four, three, six are crossed out to three, two, one?
> Anne: He definitely has copied (for the blocks) and ran out of time (for blank circles)…
> Mel: Yes and there is even the same reversal of the three and the two (pointing to the two worksheets)

We interpreted these learners to have defined the task for themselves as completing the worksheet by writing their name, colouring the trees and then copying the numerals, pictures and dots of someone else. While this interpretation seems to focus on the *product* (what the worksheet should look like when filled in) rather than on the *process* of using the worksheet to represent ones own written representation of ones sense making of the changing pattern of quantities in the story. Anne noted that there is still some skill and value in this as students count the dots of others and reproduce them and practice colouring, writing their names and copying numerals.

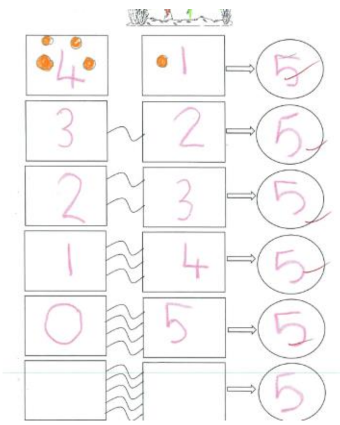
5.2.2 Interpretations of Thandi's student worksheets

All 22 of Thandi's worksheets had either numerals and dots (20) or numerals only (2) and all 22 wrote the numeral 5 in each of the six circles. Twenty began with 5-0 in the top row while two began with 4-1 in the top row. When looking at the four worksheets that only used numerals Thandi explained:

> Thandi: They just listen when I say four plus one they just write because I was not to force them (to draw bottle tops) so when I say five they just think about the number five.
> Mel: So do you think these are the stronger ones and so they didn't draw the dots because they are like aah I know?
> Thandi: No for me its because they were just thinking about the maths because I didn't sit with them to force them I said 'four plus one' they say 'is five' I say 'now write down' or because they are thinking I say 'you must write big numbers' so I don't have space to write those things (bottle tops)
> Mel: Ooh Okay that's a good insight. Was there any discussion about the lines?
> Thandi: Others could see the lines because the lines were helping them to know the steps because I saw another one was counting on those lines - he sees two then writes two (pointing to the 2 lines in the 3rd row and the written 2 in the box). For some they were not thinking about it they were just counting the lines.

Reflecting on the two student worksheets that started with 4-1 (see Fig 4 below) Thandi hypothesised that when she was saying 'five plus zero they were maybe thinking zero is nothing so I have to write nothing because I say zero so she is talking about nothing so 'I don't have to write'.

Figure 4: Example of a learner beginning at with 4-1



Thandi's use of bottle tops to model each step of the story, paired with her specifications on what students must write in each row of the task led to less variation in student responses than for Anne's class. Her specifications connected with her view that since the worksheet format was similar to picture sums students completed in departmentally issued workbooks (with + and = symbols) the aim was to get learners to work with *the sums* to 5 at each stage of the story. When asked: "What do you think learners learnt from this worksheet if anything?" Thandi replied: "They learnt plus and minus they also learnt maths.' When asked about the connector lines and arrows on the worksheet Thandi indicated that the use of + and = would have been better for her students:

> Mel: So tell me what is interesting for me is I'm not convinced by these lines or these arrows for your learners do you think it would have been better had I had plusses (for these connector lines) and equals (for the arrows)?
> Thandi: For sure I don't understand why you – let me just give you an example. In this book we have these. [Shows me in the workbooks: Block (with 1 elephant) + Block (with 1 elephant) = Block (with numeral 2); Block (with 2 kudu) + Block (with 3 kudu) = Block (with numeral 5) etc.]
> Mel: So when you were working with your learners it was plus and equals (pointing to the task's connector lines and arrows).
> Thandi: Yes that is why for my learners it was easy.
> Mel: So this plus and equals does it come in in term 1?
> Thandi: No its up to you - you can do it when you see the learners are ready but you can't let them go without it.
> Mel: So in a sense I was trying to get clever with a pre pre symbolic but then in fact these lines don't look like jumps because they are squiggles and then this is an arrow and so I think all my not wanting to use + and = in case the learners didn't know it was in fact probably a little confusing for those who were interpreting it like that.
> Thandi: Mm (in agreement)… Once you follow the book you know exactly in each term what you must teach. In term four my learners are able to do sums to ten
> Mel: But I think what I'm going to do in future is have no lines here, no arrows here and then the teacher can decide?
> Thandi: You can even put + but sometimes I don't know how other teachers do it.
> Mel: That's why I think I must leave it blank
> Thandi then shows me a page of sums by a student: 2 + 1 = 3; 2 + 2 = 4... up to 2 + 8 = 10

The latter part in the discussion points to the challenges of designing tasks that will be used by a range of teachers across a range of classroom contexts. The avoidance of using the symbols + and = was partly because Mel did not see the task of retelling the story as being about generating five different sums to 5 (although she did expect that subsequent work with sums, likely in Grade 1, could build on these combinations of splitting 5) but, in distinction, for her the focus is on the pattern of changing quantities (1 more and 1 less) as the story unfolds and how the total of 5 monkeys in both trees is constant in each stage of the story. The intention was to provoke thinking in part-part-whole terms that the five monkeys can be split between the two trees in six different ways and the story splits them in a patterned sequence. The opportunity for learners to retell and make sense of the patterned changing quantity of monkeys in each tree at each stage is constrained by the strong specifications that point learner attention at each stage to the sums. On the other hand students are able to work with these sums and are likely to connect these with other work they have done.

7. Discussion and concluding remarks

Grade R learners need clear boundaries and instructions to enable confidence but also sufficient freedom to explore sense making in their own informal way – exploring the use of this task has shown tension between specifications that can constrain (in the design, e.g., connector lines) and in the instructions given to learners. The interview with Thandi is much shorter than Anne's because there is much less variation in student work to be discussed and this results in less speculation on the part of Mel (first author) and the teacher in terms of what students may have been thinking. However the openness of Anne's introduction to the task and her not wanting to influence what they write so that she could see what they came up with created a productive space for rich reflection on both the design of the task and possible adaptations and possible student interpretations. In Anne's class we see that the absence of any verbal specification, or pictorial clue in the design, of what the circles represent, rendered the intended aspect of noting 5 as a constant total, invisible. All of Thandi's students wrote 5 in each circle making the 5 as a constant answer visible though whether students noted this as a constant total in the number of monkeys in both trees (as intended) or as a constant answer to the teachers sums (e.g. 3+2 is? 4+1 is?) cannot be inferred.

One distinction that emerged between researcher and teacher interpretations of the purpose of the task was about a focus on the constant sum (for Thandi) and, for Mel, a focus on partitioning five in different ways. A third possibility is seen in Jaya's worksheet (Figure 3) where she appears to focus on the dynamic in the story by representing the change at each stage. We do not want to place value judgments on these different purposes, or areas of focus, and each one might be appropriate for learners at different times. The point of interest is that there is this difference and what, as teachers or researchers, we do about multiple interpretations.

Of course, we expect differences of interpretation and intention between: task designer; teacher educator; teacher; student. These differences emerge particularly when task specifications are left open or even ambiguous. While there are some specifications in the layout and design of the task, Mel's suggestion that teachers use this should they wish (and should they think their learners are ready) and to use it to allow learners to retell the story was intended to allow a range of responses and uses (although the pilot example given possibly closed this space for some teachers who may have interpreted it as the memo for the worksheet). Anne uses this lack of specification of how learners should complete the task and specified very little to her class other than to use the worksheet to tell the story. In this respect some learners recast the task to be one of primarily colouring and copying numbers or pictures from others.

Thandi specifies at each step what learners should write, explicitly connecting this with sums, which learners have done and will continue to do in Grade 1. In each class students are learning about number and the point of this article is not to judge which class of learners may have gained more from the activity. However what is illuminated here is that explicit reflection on the differences of interpretation, as a result of weaker specification in tasks, is generative of much discussion (between students and teacher and between teacher and teacher educator).

We recognise that, for example, institutional demands and accountability pressures may make it seem desirable to try to communicate in an unambiguous manner to teachers or to learners. Furthermore as noted in the discussion of context above, a focus on helping students collectively to produce the 'right' answer and awarding marks (or in this case ticks and crosses) can reduce opportunities for individual sense making and progression. It might be tempting to think that if only task specifications were explicit enough, teachers and learners would learn what researchers and teachers want them to learn. And, we accept there may be times when it is desirable to constrain learner responses. In contrast, we have shown how differences in interpretation can be used productively if space is given to share the multiple views and reactions to tasks that will inevitably arise. We have shown how this space can operate both in a classroom with a teacher working on tasks with students and with a researcher, working on tasks with teachers. Students will make their own sense of tasks and this sense cannot be the same as the sense made by the teacher, and similarly with teachers and task-designers. It is possible to close off the space for discussion of these differences, or open up possibilities. Hearing the voices of the teachers was, for Mel (the researcher and task designer), a powerful learning experience in terms of allowing her access to different interpretations; and, in the classroom, allowing students to express their different interpretations allowed access to the distinctions they made within the story. If learning is the making of new distinctions then we argue that in resisting the desire for the unambiguous, we can position ourselves (researchers and teachers) as explicitly exploring differences in interpretation in order to support the learning of others and ourselves.

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